Published by Institute of Physics Publishing for SISSA/ISAS

Received: *August 25, 2004*
Revised: *November 5, 2004*
Accepted: *November 11, 2004*

Comments on 2D type-IIA string and matrix model

Tadashi Takayanagi

Jefferson Physical Laboratory, Harvard University
Cambridge, MA 02138, U.S.A.
E-mail: takayana@bose.harvard.edu

Abstract: We consider a type IIA-like string theory with RR-flux in two dimension and propose its matrix model dual. This string theory describes a Majorana fermion in the two dimensional spacetime. We also discuss its scattering amplitudes both in the world-sheet theory and in the matrix model.

Keywords: Superstrings and Heterotic Strings, Conformal Field Models in String Theory, 2D Gravity, Matrix Models.

http://jhep.sissa.it/archive/papers/jhep112004030/jhep112004030.pdf

JHEP11(2004)030

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1. Introduction

The two dimensional string theory [1] is a very useful laboratory of quantum gravity, where we can solve the theory exactly by employing the dual $c = 1$ matrix model.[1] In spite of its low dimensionality this string theory is dynamical by exciting a massless scalar field. Thus we can analyze the dynamical processes in the two dimensional quantum gravity non-perturbatively. Even though the two dimensional bosonic string turned out to be non-perturbatively unstable [6], the two dimensional type 0 string is non-perturbatively well-defined [7, 8] . This can be seen from its matrix model dual [7, 8] , obtained in the light of the recent interpretation of fermions in the matrix model as unstable D-branes [9, 10].

Usually in superstring we have two kinds of theories i.e. type II and type 0 , and we are more interested in type II than type 0 since the latter includes a closed string tachyon and no fermions in ten dimension. Thus it will be basic and important to ask whether we can construct a non-perturbatively stable type II string in two dimension, though even the type 0 theory is completely stable in two dimension. One way to define a type II string is to take a $\mathbf{Z}_2$ orbifold of type 0 string by the action $(-1)^{F_L}$ of the world-sheet fermion parity, though this theory has no spacetime supersymmetry. In type II string we cannot put a cosmological constant term to make the theory weakly coupled because it is not allowed by the GSO projection. Instead we can consider a similar Liouville-like term of

[1]For reviews see e.g. [2]–[5].

RR vertex operators. In the paper [11] a matrix dual of type IIB-like string theory was proposed following this idea (see [12, 13] for other proposals by using the supersymmetric matrix model [14]). The resulting structure is the same as that of the $c = 1$ matrix model. However, this construction seems to be non-perturbatively unstable as noted in [11] and we will probably need a refinement about its non-perturbative corrections.

Motivated by this, we would like to consider a type IIA-like string (below we will just call this a type IIA string) from the viewpoints of both the world-sheet and matrix model. We consider type 0A with RR-flux (see e.g. [15]–[18] and [11] for recent discussions) and take a $\mathbb{Z}_2$ quotient to define the type IIA string. As we will see later, the dynamical field in the IIA string is a Majorana fermion coupled to the two dimensional linear-dilaton gravity. Obviously this model is non-perturbatively well-defined as in the original 0A theory.

We organize this paper as follows. In section 2 we give a world-sheet description of two dimensional type II string. In section 3 we propose a matrix model dual of type IIA string and discuss its properties. In section 4 we compute some of tree level scattering amplitudes in the IIA string and try to compare them with the results in the dual matrix model. In section 5 we summarize the conclusions.

2. World-sheet theory of 2D type IIA string

2.1 Definition of 2D type II string

The world-sheet fields[2] in two dimensional superstring consist of the $\hat{c} = 1$ matter and the super-Liouville fields [19]–[21]. The fields in the $\hat{c} = 1$ sector are a free boson X_0 and its superpartner ψ_0. They describe the time coordinate of the two dimensional spacetime. Those in the super-Liouville sector are the Liouville field ϕ and its superpartner ψ_1. The Liouville field has the background charge $Q = 2$ (central charge $c = 1 + 3Q^2 = 13$) and describes the space coordinate with a linear dilaton $g_s = e^{\phi}$. Since the string theory becomes strongly coupled when ϕ becomes large, usually we put a (super)Liouville term

$$\mu \int dz^2 d\theta^2 e^{\Phi} , \tag{2.1}$$

in order to regulate the strongly coupled region. Then we can define physical vertex operators in NS and R sectors by

$$\begin{aligned} V_{\rm NS} &= e^{-\varphi} e^{\phi + iP(\phi \pm X^0)} , \\ V_{R(\epsilon)} &= e^{-\frac{1}{2}\varphi} e^{\epsilon \frac{i}{2} H} e^{\phi + iP(\phi + \epsilon X^0)} , \end{aligned} \tag{2.2}$$

where $\epsilon = \pm$ represents the chirality of R-sector fermion in the spacetime and comes from the two choices of ground states in R-sector. The field H is the bosonization of the two real fermions ψ_0 and ψ_1. Note that the chirality determines the traveling direction of R-sector fields due to the 'Dirac equation' or the super-Virasoro constraint $G_0|phys\rangle = 0$. We call a field with $\epsilon = +$ (or $\epsilon = -$) a left-moving (or right-moving) one in the two dimensional spacetime.

[2] In this paper we set $\alpha' = 2$.

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To define consistent string models we need GSO projections. The non-chiral GSO projections lead to the type 0A and 0B theory [7, 8] defined by the following chiral- and antichiral- sectors of closed string,

$$\begin{aligned} 0A &: (\mathrm{NS},\mathrm{NS}),(R(+),R(-)),(R(-),R(+))\,, \\ 0B &: (\mathrm{NS},\mathrm{NS}),(R(+),R(+)),(R(-),R(-))\,. \end{aligned} \tag{2.3}$$

The NSNS sector in each theory represents a massless scalar field, which was originally a tachyon field in the familiar ten dimensional type 0 string theory. The RR sector in the 0B theory corresponds to a scalar field (axion). In the 0A theory a RR vertex operator cannot have a non-zero momentum. This is because it corresponds to a one-form RR gauge potential whose field-strength is two form in the two dimensional spacetime. Its equation of motion requires that the RR-flux should be constant. The matrix model dual to these theories were given[3] in [7] [8].

In this paper we would like to consider other kinds of two dimensional string models obtained from a chiral GSO-projection. We call them type IIA and IIB, since they can also be obtained from the $\mathbb{Z}_2$ orbifold by the action $(-1)^{F_L}$ of the 0A and 0B theory. These type II models are defined by the sectors

$$\begin{aligned} IIA &: (\mathrm{NS},R(-)),(R(+),\mathrm{NS}),(R(-),R(+))\,, \\ IIB &: (\mathrm{NS},R(-)),(R(-),\mathrm{NS}),(R(+),R(+))\,. \end{aligned} \tag{2.4}$$

The chiralities of R-sectors are determined such that the OPEs between vertex operators are local with each other, which is a usual procedure to find a correct GSO projection. Earlier discussions of the related typeII-like models can be found in [21, 26, 12]. Indeed the model (2.4) we are disucssing here is equivalent to the zero radius limit $R \to 0$ of the two dimensional superstring considered in [26]. In the paper [26], the superstring model is defined to be the $\mathbb{Z}_2$ orbifold of compactified type 0 string by the $\mathbb{Z}_2$ action $(-1)^{F_L} \cdot \sigma_{1/2}$, where $\sigma_{1/2}$ denotes the half-shift $X \to X + \pi R$. Thus in the limit $R \to 0$ the model becomes[4] type II string and is the same as our non-compact model (2.4) after T-duality.

The physical fields in the IIB background are given by a left-moving RR scalar field and also a right-moving Dirac fermion in the NSR and RNS sector. Though we call this theory type II, there is no spacetime supersymmetry actually as long as the time-direction is not compactified. Applying a bosonization of the fermion in two dimension, we get a single massless scalar field after combined with the RR field. This is the same field content as the familiar two dimensional bosonic string. Indeed as argued in [11], the matrix model dual

[3] A further check of this matrix model proposal of the 2d type 0 string was given in [22] from the viewpoint of holography (or open/closed duality) by analyzing loop operators. From this analysis we can find the world-sheet supersymmetry implicitly by identifying what are the NSNS and RR sector in the matrix model side, which is the most important difference from the 2d bosonic string. For other related discussions on the interpretations of $c = 1$ or $c < 1$ matrix models via open/closed duality, refer to [9] and [23]–[25].

[4] Notice that the orbifold action (3.23) in [26] is the same as $(-1)^{F_L}\sigma_{1/2}$. Indeed we can check that the explit spectrum (3.24),(3.25) and (3.26) obtained in [26] leads to only fields in NSR and RR sectors in the limit $R \to 0$ because the mass becomes infinite in the NSNS sector. On the other hand, when $R \to \infty$, the NSR sector fields become infinitely massive as is expected in type 0 string.

of IIB can be obtained from the corresponding $\mathbb{Z}_2$ projection of type 0B model with a non-zero RR-flux in the matrix model side[5] and this has the same structure as the $c = 1$ matrix model. In this definition, a constant RR-flux plays a role of the 'Liouville term' which regulates the strongly coupled region[6] instead of the conventional cosmological constant term (2.1). Notice that the term (2.1) corresponds to a closed string tachyon condensation and the tachyon field is projected out in type II models. However, this definition of IIB string is at the perturbative level since the RR-flux background of the original type 0B theory is not non-perturbatively well defined. It is not clear how to define the matrix model dual to type IIB non-perturbatively, though we believe that should be possible.

Motivated by this we would like to turn to the IIA model[7] because the RR-flux background in the 0A model is non-perturbatively well-defined. In the IIA model, the spacetime fields in the $(R(+),\mathrm{NS})$ and $(\mathrm{NS},R(-))$ sectors are given by a Majorana left-moving and right-moving fermion $\psi(x^+)$ and $\tilde{\psi}(x^-)$. The RR field in $(R(-),R(+))$ is RR 1-form potential C (or 2-form RR field-strength F). Note that the physical state constraint $G_0|phys\rangle = 0$ kills all propagating modes and only the zero-mode is allowed as in the 0A case. Again we have the background constant RR-flux $F_{t\phi} = q$, which is represented by the vertex operator

$$S_{\mathrm{RRflux}} = q\int_\Sigma dzd\bar{z}\ V_{\mathrm{RR}} = q\int_\Sigma dzd\bar{z}\ e^{-\frac{1}{2}\varphi(z)-\frac{1}{2}\varphi(\bar{z})}e^{\frac{i}{2}H(z)-\frac{i}{2}H(\bar{z})}e^{\phi(z,\bar{z})}. \tag{2.5}$$

This includes an exponential e^ϕ factor and thus regulates the strongly coupled region at $\phi \sim -\log q$. The large q means that the theory is weakly coupled. The effective description by the extremal black-hole solution [28] will also be applied to this IIA model with RR-flux as in the 0A model discussed in [18] [11] (see also [29, 30] for further discussions).

2.2 Minisuperspace approximation

As we have seen, the background RR-flux $F_{t\phi} = q$ makes the IIA theory weakly coupled. It will lead to an effective potential wall in the same way as (2.1). The reflection of a propagating field due to such a wall can be usually well described by the minisuperspace approximation as has been done in bosonic string [31] and type 0 string [8]. Since the RR-vertex operator is also proportional to e^ϕ (Liouville dressing) in our case, we may expect the minisuperspace approximation is given by the action like

$$S = \int dtd\phi\left[i\psi\partial_+\psi + i\tilde{\psi}\partial_-\tilde{\psi} + 2iqe^\phi\psi\tilde{\psi}\right], \tag{2.6}$$

where $x^\pm = \frac{1}{2}(t\pm\phi)$. The Majorana fermions ψ and $\tilde{\psi}$ correspond to the left-moving and right-moving part in the asymptotic region $\phi\to-\infty$, respectively. This can be understood if we assume the (Lorentz invariant) coupling $\bar{\Psi}F_{\mu\nu}\Gamma^{\mu\nu}\Gamma^{01}\Psi$ between the fermions and

[5]See also [12] for another proposal for a N=2 Liouville NSNS background using a supersymmetric matrix model [14] from the viewpoint of multiples D-branes. The D-branes in the N=2 Liouville theory were classified in [27].

[6]More precisely, we should say that the Liouville like term consists of RR-flux and also bosonized field of NSR fermion as discussed in [11].

[7]A classical Green-Schwarz string of type IIA string AdS_2 was proposed recently in [13] by using a supercoset (see also [28]).

RR-fields. We can also find the similar behavior from the matrix model side discussed later. We can solve the wavefunction of (2.6) exactly as follows. The equation of motion is given by

$$\partial_+\psi + 2q\tilde{\psi}e^{\phi} = 0\,, \qquad \partial_-\tilde{\psi} - 2q\psi e^{\phi} = 0\,. \tag{2.7}$$

The Majorana fermions ψ and $\tilde{\psi}$ with energy ω satisfy the differential equations ($l = e^{\phi}$)

$$\begin{aligned}(l^2\partial_l^2 + \omega^2 + i\omega - 4q^2l^2)\psi &= 0\,,\\ (l^2\partial_l^2 + \omega^2 - i\omega - 4q^2l^2)\tilde{\psi} &= 0\,.\end{aligned} \tag{2.8}$$

Also we require that in the limit $\phi \to +\infty$ they should decay exponentially due to the mass term or Liouville like potential. Thus we find the solutions up to a normalization

$$\begin{aligned}\psi(t,\phi) &= e^{-i\omega t}e^{\phi/2}K_{i\omega-1/2}(2qe^{\phi})\,,\\ \tilde{\psi}(t,\phi) &= e^{-i\omega t}e^{\phi/2}K_{i\omega+1/2}(2qe^{\phi})\,.\end{aligned} \tag{2.9}$$

Indeed they behave in the strongly coupled region $\phi \to +\infty$ as

$$\tilde{\psi}(t,\phi) \sim \psi(t,\phi) \sim \frac{\pi}{4q}e^{-i\omega t}e^{-qe^{\phi}}\,. \tag{2.10}$$

On the other hand in the weakly coupled region $\phi \to -\infty$,

$$\begin{aligned}\psi(t,\phi) &= \frac{1}{2}q^{i\omega-1/2}\Gamma\left(\frac{1}{2} - i\omega\right)\, e^{-i\omega(t-\phi)}\,,\\ \tilde{\psi}(t,\phi) &= \frac{1}{2}q^{-i\omega-1/2}\Gamma\left(\frac{1}{2} + i\omega\right)\, e^{-i\omega(t+\phi)}\,.\end{aligned} \tag{2.11}$$

From this we find the reflection amplitude of NSR fermion in the minisuperspace approximation

$$S(\omega)^{ms}_{\mathrm{NSR}} = q^{-2i\omega}\frac{\Gamma\left(\frac{1}{2}+i\omega\right)}{\Gamma\left(\frac{1}{2}-i\omega\right)}. \tag{2.12}$$

Interestingly, this result is the same as the minisuperspace approximation of the RR scalar field in the type 0B matrix model computed in [8].

2.3 Discrete states

In addition to the continuous state (2.2), there are also another kind of physical states at discrete imaginary momenta. This is a special property of two dimensional string theories. They are called discrete states and are computed in [32, 33] in the $\hat{c} = 1$ case. In a chiral sector they are given by (for positive r and s)

$$|W_{(r,s)}\rangle = \left(\int \psi e^{-iX}\right)^s\, e^{\frac{i}{2}(r+s)X}e^{\frac{1}{2}(r+s+2)\phi}|0\rangle\,, \tag{2.13}$$

in the euclidean theory ($X = iX_0$), where r and s are integers which satisfy $rs > 0$. The states for negative r and s can also be obtained in a similar way by a dual operation. The state $|0\rangle$ represents the ground state i.e. the -1 picture vacuum for the NS sector

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and -1/2 picture vacuum with the negative chirality for the R sector. Discrete states with $r-s \in 2\mathbb{Z}$ (or $r-s \in 2\mathbb{Z}+1$) belong to the NS-sector (or R-sector). Notice that this fact is consistent with the position of poles of the NSNS and RR leg factor $e^{\delta_{\rm NSNS}(p)} \propto \frac{\Gamma(iP)}{\Gamma(-iP)}$ and $e^{\delta_{RR}(P)} \propto \frac{\Gamma(1/2+iP)}{\Gamma(1/2-iP)}$ [7, 8], respectively. When we consider the chiral and antichiral sector to define a closed string theory, the momentum of ϕ should be the same in both sectors. Thus the discrete states appear only in NSNS and RR sectors. Furthermore, since here we assume that the time coordinate is also non-compact, the momentum of X should also be the same in the chiral and antichiral sector.

The 0B theory includes all of the NSNS and RR discrete states in the left-right symmetric way. For example, a massive graviton appears in the NSNS discrete sates. In the 0A theory discrete states only exist in the NSNS sector. There are no RR-sector ones because of the left-right asymmetric GSO projection. In a similar way we can also define the discrete states in the type IIA and IIB model. Both can be obtained after the $\mathbb{Z}_2$ twist of the 0A and 0B by the operator $(-1)^{F_L}$. In IIB theory they are given by $(r,s) \in (2\mathbb{Z}+1, 2\mathbb{Z}+1)$ for the NSNS-sector and $(r,s) \in (2\mathbb{Z}, 2\mathbb{Z}+1)$ for the RR-sector. In the IIA model we get $(r,s) \in (2\mathbb{Z}+1, 2\mathbb{Z}+1)$ for the NSNS-sector and none for the RR-sector. Note that there are no twisted sectors (or equally the NSR and RNS sector) as there are no discrete states in the NSR or RNS sector.

Finally we would like to briefly examine the ground ring structure of these string models. We define the generators of the ground ring by x and y. Each of them is a BRST invariant operator with ghost number zero including both chiral and antichiral part symmetrically. For the precise definition of these operators see [34, 8]. In the type 0B the ground ring is generated by x and y, while in the 0A it is generated by x^2, y^2 and xy. The discrete state $W_{(r,s)}$ corresponds to the ground ring element $x^{-r-1}y^{-s-1}$ [34]. The ground ring structure of type 0 string was discussed from geometrical viewpoints in [35]. We would also like to apply this argument to the type II models. The ground ring of IIA is generated by x^2 and y^2, while that of IIB by x and y^2. This result can be easily understood as the $\mathbb{Z}_2$ projection $y \to -y$, which is equivalent to $(-1)^{F_L}$. Generally the ground ring structure represents the W_∞ symmetry of matrix model [36]. As we will see later, the above results on type II string can be correctly reproduced from the proposed matrix model.

3. A proposal of matrix model dual

3.1 IIA matrix model

Now we would like to construct a matrix model dual of the type IIA string in two dimension, whose properties on the world-sheet have been discussed in the previous section. We argue that the IIA matrix model can be obtained by the $\mathbb{Z}_2$ projection $(-1)^{F_R} = -(-1)^{F_L} = 1$ of the type 0A model as in the world-sheet theory.[8] The similar construction has already proposed for IIB model in [11]. The IIB model is defined by the $\mathbb{Z}_2$ projection $(-1)^{F_L} = 1$ of the 0B matrix model. This $\mathbb{Z}_2$ action can be identified with the transformation of a

[8] A matrix model dual of IIA string in AdS_2 was proposed in [13]. As is clear from section 2.3, our model does not have any super-W_∞ symmetry, while the AdS_2 model seems to have it.

fermion (or hole) into a hole (or fermion) and the operation $(x,p) \to (p,x)$ at the same time [8]. The cosmological constant changes its sign under this action. We will apply the similar method to define the IIA model.

The 0A matrix model [8, 7] is equivalent to the hermitean matrix model with the following deformed hamiltonian [37] as shown in [15]

$$2H = p^2 - x^2 + \frac{M}{x^2}\,, \qquad M \equiv q^2 - \frac{1}{4}\,. \tag{3.1}$$

In addition to the hamiltonian, we have the following conserved charges [38] almost in the same way as the W_∞ charge in the $c=1$ matrix model

$$\begin{aligned} W_+ &= e^{-2t}\left((p+x)^2 + \frac{M}{x^2}\right) = 2\sqrt{M+\mu^2}\,, \\ W_- &= e^{2t}\left((p-x)^2 + \frac{M}{x^2}\right) = 2\sqrt{M+\mu^2}\,, \end{aligned} \tag{3.2}$$

for the classical trajectory $x^2(t) = \mu + \sqrt{M+\mu^2}\cosh(2t)$. Any general conserved charges can be written by the product of W_+, W_- and H summed over each fermions [38]. In the 0A theory we can also define the $\mathbb{Z}_2$ action by the combination of the hole-particle exchange and the action

$$x' = \sqrt{p^2 + \frac{M}{x^2}}\,, \qquad p' = \frac{px}{\sqrt{p^2 + M/x^2}}\,. \tag{3.3}$$

Under this transformation, we can show $dx' \wedge dp' = -dx \wedge dp$ and thus this is a canonical transformation. This extra minus sign, which also appears in the 0B case (just setting $M=0$), is due to the fact that the hole has a minus momentum compared with a particle. Under this $\mathbb{Z}_2$ action, the hamiltonian changes its sign $H \to -H$ (or equally $\mu \to -\mu$), while the other charges remain unchanged $W_\pm \to W_\pm$. These facts are consistent with our previous analysis on the ground ring structure. As in the usual $c=1$ matrix model, the three ground ring generators x^2, y^2 and xy of 0A model are naturally identified with W_+, W_- and H, respectively. Only the operator xy changes its sign under the $\mathbb{Z}_2$ action by $(-1)^{F_L}$ and it is projected out in IIA model. This exactly agrees with its action in our matrix model side. The same argument can also be applied to the matrix model dual of IIB theory [11]. We can also see this in a non-perturbative way by applying the exact wave function [39] to the quantum mechanics for (3.1). Indeed the density of state[9] for the fermions is $\mathbb{Z}_2$ symmetric $\rho(\epsilon, q) = \rho(-\epsilon, q)$ [11]. Then the non-perturbative $\mathbb{Z}_2$ action can be written as

$$a_\epsilon \to a^\dagger_{-\epsilon}\,, \tag{3.4}$$

where we denote the creation and annihilation operator of fermion with the energy ϵ by $a^\dagger_\epsilon,\ a_\epsilon$.

From the above arguments, at $\mu = 0$ we have the enhanced $\mathbb{Z}_2$ symmetry[10] and we can indeed take a quotient by this action. We would like to argue that this defines the

[9] This can be computed from the phase shift of the wave function.

[10] This should be the origin of the $\mathbb{Z}_2$ symmetry in the tachyon scattering amplitudes found in [37] (see also [38, 39, 11]) as can be seen from the action $\mu \to -\mu$.

IIA matrix model. After we factor out the semiclassical part of the wave function by the double-scaling limit, we get a free relativistic Dirac fermion in the asymptotic region as in the $c = 1$ case [40, 2] before the $\mathbb{Z}_2$ projection

$$\Psi_L = \sum_{n\in\mathbb{Z}} a_n^L e^{in\omega_0(\tau-t)}\,, \qquad \Psi_R = \sum_{n\in\mathbb{Z}} a_n^R\; e^{-in\omega_0(\tau+t)}\,, \tag{3.5}$$

where the 'spacial' coordinate $\tau(\sim\phi)$ is defined by $x \sim \sqrt{2\mu}\cosh(\tau)$. Here we put a cutoff for a large value of $|\tau|$ and that leads to a discrete energy $n\omega_0$. The $\mathbb{Z}_2$ projection identifies a_n with $a^\dagger_{-n}$. After this identification, the fermion becomes real

$$\Psi^\dagger_{L,R}(\tau,t) = \Psi_{L,R}(\tau,t)\,. \tag{3.6}$$

After we take the boundary condition $a_n^L = a_n^R$ into account, the only dynamical field in the spacetime is a Majorana fermion. Indeed, this is the same conclusion as in the previous world-sheet analysis.

3.2 Scattering amplitude from matrix model

The action of the fermion field in the 0A theory is roughly given by the usual kinetic term of Dirac fermion plus position dependent Dirac mass term $\sim \frac{1}{\sqrt{M}}e^{-2\tau}(\Psi_L^\dagger\Psi_L + \Psi_R^\dagger\Psi_R)$ [41, 40]. The point is that the left and right-moving sector are completely decoupled with respect to the coordinate (t,τ). Naively one may think there is no scattering in such a background of string theory. However, the space coordinate τ, which is defined by $x^2 = \mu + \sqrt{M+\mu^2}\cosh(2\tau)$, is non-locally related to usual space coordinate ϕ in the string theory as is well-known. In the asymptotic region we have $\phi \sim -|\tau|$. Thus a fermion which propagates from $\tau = -\infty$ to $\tau = \infty$ describes a fermion scattered off the Liouville potential in string theory. This behavior of a Majorana fermion agrees with the minisuperspace action (2.6) and the results in section 2.2 at least qualitatively.

In the IIA model, since we have the Majorana projection (3.6), we have the simple fermion action

$$S = \int dt d\tau (\Psi_L(\partial_t + \partial_\tau)\Psi_L + \Psi_R(\partial_t - \partial_\tau)\Psi_R)\,. \tag{3.7}$$

Even though this is free, the non-local transformation leads to a reflection at the Liouville potential as we have explained just before.

In the exact wave function analysis of [39] we can get the non-perturbative S-matrix (or the reflection amplitude) for energy ω (in $\alpha' = 2$ convention)

$$R_\omega = \left(\frac{4}{q^2 - \frac{1}{4}}\right)^{-i\omega} \frac{\Gamma(\frac{1}{2} - i\omega + \frac{|q|}{2})}{\Gamma(\frac{1}{2} + i\omega + \frac{|q|}{2})} e^{i\pi q/2}\,. \tag{3.8}$$

Since this is a pure phase factor, we can conclude that an incoming RNS fermion is completely reflected at the wall and the fermion number is conserved. Notice that this scattering amplitude of fermions takes a rather different form than those of bosons in bosonic and type 0 string [42, 43].

To be consistent with the $\mathbb{Z}_2$ projection (3.6) we should have $R^*_\omega = R_{-\omega}$. This gives an intriguing quantization $q \in 2\mathbb{Z}$, which implies that the odd number $(= q)$ of D0-branes will not be consistent with the $\mathbb{Z}_2$ orbifold.[11] In this case the last factor $e^{i\pi|q|/2}$ is just ± 1. We can show that the expansion of (3.8) for the large $|q|$ (or weak coupling) is of the form

$$R_\omega = \pm(1 + \sum_{n=1}^{\infty} r_n(\omega) q^{-2n})\,. \tag{3.9}$$

This can be seen from the relation $\frac{S(-|q|)}{S(|q|)} = \frac{\sin(\frac{1}{2} - \frac{i\omega}{2} + \frac{|q|}{2})}{\sin\pi(\frac{1}{2} + \frac{i\omega}{2} + \frac{|q|}{2})}$, where $S(|q|) \equiv \frac{\Gamma(\frac{1}{2} - \frac{i\omega}{2} + \frac{|q|}{2})}{\Gamma(\frac{1}{2} + \frac{i\omega}{2} + \frac{|q|}{2})}$. After we neglect the non-perturbative part like $\sim e^{i|q|}$, we get the expansion (3.9). In this way we can see the expected perturbative expansion of closed string with respect to the string coupling $g_s^2 \sim q^{-2}$. This is non-trivial since in the bosonic or type0 string we regard a fermion as a D0-brane [7]–[10] for which it is in principle possible to have a $g_s \sim q^{-1}$ expansion.

4. S-matrix of 2D type IIA string

The scattering S-matrix should be one of the most basic quantities when we compare a string theory with its dual matrix model. The dynamical field in the IIA model is a Majorana fermion as we have observed in both the world-sheet theory (section 2) and its matrix model dual (section 3). Thus it will be useful to compare the scattering amplitudes in both sides. Since we have done in the matrix model side in section 3.2, here we would like to analyze those in the world-sheet computations. Similar computations have been done in [21] [44] for the type 0 string. In this section we compute the scattering amplitudes of NSR(or RNS) fermions following the method and conventions[12] in [21]. Amplitudes can be written in the following form (notice also that on-shell NSR(RNS) vertex operators represent incoming(outgoing) fermions)

$$A(p_1, \dots, p_{M+N}) = q^s \left\langle (V_{\mathrm{NSR}})^N \quad (V_{\mathrm{RNS}})^M \quad \left(\int V_{RR}\right)^s \right\rangle, \tag{4.1}$$

for $N \to M$ scattering with s insertions of Liouville-like term or RR-flux term given by (2.5). The non-negative integer s is determined such that the sum of all ϕ momenta of the RNS or NSR vertex operators is given by $-Q - s$. Even though s becomes a non-integer value for general momenta, it is natural to believe that the general amplitudes are given by an analytical continuation as usual in Liouville theory. However, it is not easy to compute the amplitudes (4.1) for any $s \in \mathbb{Z} \geq 0$ in a systematical way due to the presence of the superghost φ and the picture changing.

[11]This may suggest that the original fermion in the 0A model can be regarded as a sort of a 'fractional brane' in IIA. This fractional object may be related to the spin operator from the viewpoint of Ising model as the IIA model includes a free Majorana fermion in the asymptotic region.

[12]Notice that the paper [21] use the $\alpha' = 2$ unit and the Liouville field ϕ corresponds to $-\phi$ in our notation defined in section 2.

Thus let us first concentrate on the amplitudes which do not include any insertion of the Liouville-like term (2.5) (i.e. $s = 0$). Since the three point function is zero due to the fermionic statics, let us compute the four point function, for example. To match the picture we consider the correlation function of the following four vertex operators

$$\left\langle V_{R(-),\mathrm{NS}}^{(-1/2,-1)}(z_1,\bar{z_1}) \cdot V_{R(-),\mathrm{NS}}^{(-1/2,0)}(z_2,\bar{z_2}) \cdot V_{\mathrm{NS},R(+)}^{(-1,-1/2)}(z_3,\bar{z_3}) \cdot V_{\mathrm{NS},R(+)}^{(0,-1/2)}(z_4,\bar{z_4}) \right\rangle . \tag{4.2}$$

We define the (euclidean) momenta of each vertex operators by $\vec{p}^i = (p_x^i, p_\phi^i) = (k_i, \beta_i)$ for $i = 1,2,3,4$. The on-shell ($L_0 = 1$) condition is $\beta_i + \frac{Q}{2} = |k_i|$ and the momentum conservation is $\sum_i k_i = 0$ and $\sum_i \beta_i = -Q$. Also note that the superconformal invariance $G_0 = 0$ requires $k_1, k_2 < 0$ and $k_3, k_4 > 0$ (called kinematical region). After we integrate the moduli z^i with the gauge fixing, we get the following amplitude

$$A(p^1,p^2,p^3,p^4) = -2\pi^4 \left(\frac{1}{2} + \vec{p}_2 \cdot \vec{p}_4\right) \frac{\Gamma(\vec{p}_1 \cdot \vec{p}_4 + \frac{1}{2})\Gamma(\vec{p}_2 \cdot \vec{p}_4 + \frac{1}{2})\Gamma(\vec{p}_3 \cdot \vec{p}_4 + 1)}{\Gamma(-\vec{p}_1 \cdot \vec{p}_4 + \frac{1}{2})\Gamma(-\vec{p}_2 \cdot \vec{p}_4 + \frac{1}{2})\Gamma(-\vec{p}_3 \cdot \vec{p}_4)} . \tag{4.3}$$

Notice that this expression is antisymmetric[13] with respect to $(\vec{p}_1, \vec{p}_2)$ and $(\vec{p}_3, \vec{p}_4)$ being consistent with their fermionic statistics. In the kinematical region $k_1, k_2 < 0$ and $k_3, k_4 > 0$, we can show that $\vec{p}_3 \cdot \vec{p}_4 = 0$. Since we have a divergence from the denominator and other factors remain finite, we can conclude that that the amplitude is zero. It is very natural that this result should extend to higher point functions. Indeed we can show this from a viewpoint of ground rings[14] applying the arguments [45]. Then we can find that the $s = 0$ amplitude is non-zero if and only if we consider $1 \to M$ or $N \to 1$ scattering. For example, if we return to our previous example $2 \to 2$ scattering, this is obviously zero in this argument. Also generally due to the fermionic statistic on the world-sheet, all of the $1 \to M$ and $N \to 1$ scatterings are zero. Thus we have found that all non-trivial S-matrices are zero if we consider $s = 0$ amplitudes or equally the linear dilaton background ($q = 0$).

Now we take the Liouville perturbation (2.5) into account. We again consider the four particle scattering for a positive integer s. This was zero when $s = 0$ as we have seen. Let us study the simplest non-trivial example of $1 \to 3$ scattering for $s = 1$. We assume the momenta of four particles are given by $k_1 > 0, k_2 < 0, k_3 < 0, k_4 < 0$. The on-shell condition and momentum conservation require $k_1 = 3/2$ and $k_2 + k_3 + k_4 = -3/2$ at the background charge $Q = 2$ for the two dimensional string theory. Then we get the following integral expression of the amplitude

$$\begin{aligned} &\int dz^2 \left\langle V_{\mathrm{NSR}}(\vec{p}_1)(0) \cdot V_{\mathrm{RNS}}(\vec{p}_2)(1) \cdot V_{\mathrm{RNS}}(\vec{p}_3)(\infty) \cdot V_{\mathrm{RNS}}(\vec{p}_4)(z) \cdot \left(\int dw^2 V_{\mathrm{RR}}(w,\bar{w})\right)\right\rangle = \\ &= -\frac{Q(2k_1 - Q/2)}{4\sqrt{2}} \left(\left(2k_4 + \frac{Q}{2}\right) I_1(k_2,k_4) + \left(2k_2 + \frac{Q}{2}\right) I_2(k_2,k_4)\right), \end{aligned} \tag{4.4}$$

[13] To see this note the identities $\vec{p}_2 \cdot \vec{p}_4 = -\vec{p}_1 \cdot \vec{p}_4 = \vec{p}_1 \cdot \vec{p}_3 = \dots$.

[14] For this we can act the ground ring elements x and y on the physical vertex operators up to the BRST cohomology. Especially for the (NS,R) sector vertex, which we are interested in, we get $x \cdot V_{\mathrm{NS},R}(q) \sim q^2 V_{R,\mathrm{NS}}(q + \frac{1}{2})$ and $y \cdot V_{\mathrm{NS},R} \sim 0$, and also an opposite relation for the RNS vertex. We can move the position of the operators x or y so that it annihilates a vertex operator.

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where we have defined the integrals as

$$
\begin{aligned}
I_1(k_2,k_4) &= \int dz^2 dw^2 |z|^{2\vec{p}_1\cdot\vec{p}_4-1}|1-z|^{2\vec{p}_2\cdot\vec{p}_4}|w|^{Q\beta_1+1}|1-w|^{Q\beta_2-1}|z-w|^{Q\beta_4-1} \\
&= \int dz^2 dw^2 |z|^{4k_4}|1-z|^{2k_3+1}|w|^2|1-w|^{-2k_2-3}|z-w|^{-2k_4-3}\,, \\
I_2(k_2,k_4) &= \int dz^2 dw^2 z^{\vec{p}_1\cdot\vec{p}_4-1/2}\bar{z}^{\vec{p}_1\cdot\vec{p}_4+1/2}|1-z|^{2\vec{p}_2\cdot\vec{p}_4} w^{Q\beta_1/2-1}\bar{w}^{Q\beta_1/2}\times \\
&\quad\times|1-w|^{Q\beta_2-1}|z-w|^{Q\beta_4-1} \\
&= \int dz^2 dw^2 z^{2k_4}\bar{z}^{2k_4+1}|1-z|^{2k_3+1}w|1-w|^{-2k_2-3}|z-w|^{-2k_4-3}\,.
\end{aligned}
\tag{4.5}
$$

In the computation of the amplitude (4.4) we chose the pictures of the five vertex operators as $(0,-1/2),(-1/2,-1),(-1/2,0),(-1/2,0)$ and $(-1/2,-1/2)$, respectively.

It is possible to perform these integrals by using the integration formula found in [46] in terms of the generalized hypergeometric function. As we show the detailed computations in the appendix A, the scattering amplitude turns out to be vanishing. However, in this case we cannot easily conclude that the S-matrix is zero for $s=1$ because we may expect the renormalization of 'cosmological constant' q. Indeed in the usual c=1 string, the naive $s>0$ amplitudes are all zero and they become finite after the renormalization $\mu_{ren}=\lim_{\epsilon\to0}\mu\epsilon$. To examine this correctly we need to regularize the amplitude. We may start with a general background charge $Q\neq2$ and a (imaginary) background charge in the X direction and then take the $Q=2$ limit as we usually do in other two dimensional string models [47, 21]. Since the analysis of the integral in this limit is very difficult, here we want to be satisfied by the evaluation of only the first integral I_1 in (4.5). Interestingly, we can show that I_1 is the same amplitude of $1\to3$ scattering with $s=1$ (in $\alpha'=1$ convention in bosonic string) if we replace k_i with $k_i+1/2$ for $i=1,2,3$. It is just a constant c times the leg-factor $\prod_i\frac{\Gamma(|k_i|)}{\Gamma(-|k_i|)}$ after the renormalization [21]. Since we know the answer in the two dimensional bosonic string (by using the matrix model computation), we get the result

$$
I_1(k_2,k_4)=cq^s\frac{\Gamma(-k_2+1/2)}{\Gamma(k_2-1/2)}\frac{\Gamma(-k_3+1/2)}{\Gamma(k_3-1/2)}\frac{\Gamma(-k_4+1/2)}{\Gamma(k_4-1/2)}. \tag{4.6}
$$

Even though we did not find the complete total expression for the amplitude, in order to see its structure this is enough. For example we can easily speculate from (4.6) the leg-factor in IIA theory

$$
e^{i\delta(\omega)}=q^{-2i\omega}\frac{\Gamma(\frac{1}{2}+i\omega)}{\Gamma(\frac{1}{2}-i\omega)}\,, \tag{4.7}
$$

and this agrees with the previous expectation from the minisuperspace computation (2.12). As the scattering amplitudes of more than four particles are much more complicated, we we will not compute explicitly in this paper. The three particle scattering is obviously zero due to the fermionic statics.

Finally we discuss the two point function. The trivial two point function like $\langle V_{\mathrm{NSR}}V_{\mathrm{NSR}}\rangle$ and $\langle V_{\mathrm{RNS}}V_{\mathrm{RNS}}\rangle$ (trivial $1\to1$ scattering) is obviously non-zero since this is the norm of the state[15] in CFT. Also the non-trivial two point function (or reflection

[15]To see this clearly it will be better to move into lorentzian frame via $|k|\to-iE$.

amplitude) is non-zero as we can be seen from the simplest case $s = 1$ of the amplitude

$$\left\langle V_{\rm NSR}\left(k, \beta = k - \frac{Q}{2}\right) \cdot V_{\rm RNS}\left(-k, \beta = k - \frac{Q}{2}\right) \left(\int dz^2 V_{\rm RR}(z, \bar{z})\right)^s \right\rangle, \tag{4.8}$$

which is obviously non-zero. For general positive integer s the amplitude is non-vanishing iff s is an odd integer. Thus we find the position of poles

$$k = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots . \tag{4.9}$$

These positions of poles agree with the previous results (2.12) and (4.6).

Before we finish this section, let us summarize the results obtained in our analysis of string scattering amplitudes. In the linear dilaton background $q = 0$ we have seen that all amplitudes except the (trivial) two point function are zero. Notice that this result is different from that of the bosonic or type 0 model, where we have non-trivial scatterings even when $\mu = 0$. In the presence of a non-zero RR-field q, we can find two possibilities: (i) all amplitudes except the two point reflection amplitude are zero, or (ii) most of the amplitudes become non-trivial as in the two dimensional bosonic or type 0 string. On the other hand, we know that in the proposed IIA matrix model dual, the fermions are free and only the reflection amplitude ($1 \to 1$ scattering) is non-trivial. Thus the transformation from a fermion ψ_{mat} in the matrix model to a fermion ψ_{IIA} in IIA string is trivial in the first case (i). In the second case (ii), however, this will become a complicated non-local transformation written, for example, in the following form

$$\psi_{mat} \sim e^{i\delta(\omega)}\psi_{IIA} + a(\omega)\psi_{IIA}\partial\psi_{IIA}\partial^2\psi_{IIA} + \cdots , \tag{4.10}$$

which changes the fermion number in addition to the presence of the leg factor. This is similar to the bosonization of fermions that appear in the bosonic or type 0 string case.

Even though we found that the $1 \to 3$ scattering amplitude with $s = 1$ is zero, a more careful analysis assuming a possible renormalization of q suggests the second possibility (ii). This was because the structures of this amplitude looks similar to that of two dimensional bosonic or type 0 string. This possibility is also more natural since we usually expect back-reactions from the gravity sector which will lead to the non-zero four point functions.[16] However, the above arguments are not conclusive and both possibilities may be possible within our results. In order to go beyond this we need a more systematic method to compute the scattering amplitudes in the presence of RR-flux.

5. Conclusions

In this paper we discussed the type IIA string theory in two dimension. We presented both the world-sheet description and its matrix model dual. This model describes a two dimensional spacetime with a Majorana fermion coupled to gravity. The matrix model description shows that it is non-perturbatively stable. We argued that this model can

[16]We would like to thank A. Strominger for pointing out this point.

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be weakly coupled due to the background RR-flux instead of the familiar cosmological constant. We also examined tree level scattering amplitudes on the world-sheet theory and compared them with the matrix model. Even though we did not determine them completely in the world-sheet computations, we found intriguing structures in the IIA amplitudes, which cannot be found in the bosonic or type 0 string. We will leave further analysis of the scattering amplitudes for a future problem. In order to compute such physical quantities in a systematical way, a construction of Green-Schwarz-like formalism may be useful. It will also be an interesting question how D-branes are related to this matrix model.

Acknowledgments

I would like to thank A. Strominger for extremely helpful comments and suggestions. I am also grateful to T. Eguchi, D. Gaiotto, D. Ghoshal, S. Gukov, J. Karczmarek, E. Martinec, J. McGreevy, S. Minwalla, S. Mukhi, Y. Nakayama, Y. Sugawara, H. Takayanagi, S. Terashima, N. Toumbas and S. Yamaguchi for useful discussions. The work was supported in part by DOE grant DE-FG02-91ER40654.

A. The proof of the vanishing of the four point amplitude

Here we would like to show that the direct evaluations of integrals I_1 and I_2 in (4.4) are zero. First let us prove explicitly that

$$I_2(k_2,k_4) = \int dz^2 dw^2 z^{2k_4}\bar{z}^{2k_4+1}|1-z|^{2k_3+1} w|1-w|^{-2k_2-3}|z-w|^{-2k_4-3}\,, \qquad \text{(A.1)}$$

is vanishing. The similar direct computation can be applied to show $I_1 = 0$ as one can easily see.

The more general form of the integral

$$I = \int dz^2 dw^2 z^{\alpha_1}(1-z)^{\alpha_2}\bar{z}^{\bar{\alpha_1}}(1-\bar{z})^{\bar{\alpha_2}} w^{\alpha'_1}(1-w)^{\alpha'_2}\bar{w}^{\bar{\alpha'_1}}(1-\bar{w})^{\bar{\alpha'_2}}|z-w|^{4\sigma}\,, \qquad \text{(A.2)}$$

was computed in [46] (see appendix of that paper) in terms of the generalized hypergeometric function ${}_3F_2$. The result of the integral is given by (we use the expression found in [48])

$$I = D_1 C^{12}(\alpha)C^{12}(\bar{\alpha}) + D_2 C^{21}(\alpha)C^{21}(\bar{\alpha}) + D_3\{C^{12}(\alpha)C^{21}(\bar{\alpha}) + C^{21}(\alpha)C^{12}(\bar{\alpha})\}\,, \qquad \text{(A.3)}$$

where $C^{ab}(\alpha)$ $(a,b=1,2,3)$ is defined by ($C^{ab}(\bar{\alpha})$ can be obtained by replacing α_a and α'_b with $\bar{\alpha}_a$ and $\bar{\alpha'}_b$)

$$\begin{aligned} C^{ab}(\alpha) = {} & \frac{\Gamma(1+\alpha_a+\alpha'_a-k')\Gamma(1+\alpha_b+\alpha'_b-k')\Gamma(1+\alpha'_a)\Gamma(1+\alpha_b)}{\Gamma(\alpha'_a-\alpha_c+1)\Gamma(\alpha_b-\alpha'_c+1)}\,\cdot \\ & \cdot {}_3F_2(1+\alpha'_a, 1+\alpha_b, k'-\alpha_c-\alpha'_c; \alpha'_a-\alpha_c+1, \alpha_b-\alpha'_c+1; 1)\,. \end{aligned} \qquad \text{(A.4)}$$

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We also defined α_3 and k' by $\alpha_1+\alpha_2+\alpha_3+1=k'=-2\sigma-1$. The functions D_i are given by

$$\begin{aligned} D_1 &= \frac{s(\alpha_1')s(\alpha_2')\left(s(\alpha_1)s(\alpha_1')s(\alpha_3)-s(\alpha_3')s(\alpha_1-k')s(\alpha_3-\alpha_2')\right)}{s(\alpha_3)s(\alpha_3')s(\alpha_3+\alpha_3'-k')}, \\ D_3 &= -\frac{s(\alpha_1)s(\alpha_1')s(\alpha_2)s(\alpha_2')s(\alpha_3+\alpha_3')}{s(\alpha_3)s(\alpha_3')s(\alpha_3+\alpha_3'-k')}, \end{aligned} \tag{A.5}$$

where $s(\alpha)\equiv\sin(\pi\alpha)$. D_2 can be obtained by replacing α_a with α_a'. The integral (A.1) corresponds to the values $\alpha_1=\bar{\alpha_1}-1=2k_4,\ \alpha_2=\bar{\alpha_2}=k_3+1/2,\ \alpha_1'=\bar{\alpha_1'}-1=0,\ \alpha_2'=\bar{\alpha_2'}=-k_2-3/2,\ k'=k_4+1/2$. Since α_1' is an integer, we can find $D_1=D_3=0$. Thus we have only to compute $C^{21}(\alpha)$ and $C^{21}(\bar{\alpha})$. $C^{21}(\alpha)$ can be obtained as follows

$$\begin{aligned} C^{21}(\alpha) &= \frac{\Gamma(2k_4+1)\Gamma(2k_3+1)\Gamma(-k_2-1/2)\Gamma(-k_4-1/2)}{\Gamma(-2k_2-1)\Gamma(k_3+1/2)}\cdot \\ &\quad\cdot {}_3F_2\left(-k_2-\frac{1}{2},2k_3+1,-1;-2k_2-1,k_3+\frac{1}{2};1\right). \end{aligned} \tag{A.6}$$

As the one of the parameters of ${}_3F_2$ is a negative integer -1, the hypergeometric function is just a sum of two terms. Easily we can see

$${}_3F_2\left(-k_2-\frac{1}{2},2k_3+1,-1;-2k_2-1,k_3+\frac{1}{2};1\right)=1-\frac{(k_2-\frac{1}{2})(2k_3-1)}{(2k_2-1)(k_3-\frac{1}{2})}=0\,. \tag{A.7}$$

Thus we have shown $C^{21}(\alpha)=0$. In the same way we can compute $C^{21}(\bar{\alpha})$ and we find a finite value

$$C^{21}(\bar{\alpha})=\frac{\Gamma(2k_4+2)\Gamma(2k_3+1)\Gamma(-k_4-1/2)\Gamma(-k_2-1/2)}{\Gamma(-2k_2)\Gamma(k_3+1/2)}. \tag{A.8}$$

In conclusion we have found $I_2(k_2,k_4)=D_2C^{21}(\alpha)C^{21}(\bar{\alpha})=0$.

Another integral I_1 in (4.4) can also be computed in a similar way. Indeed we can see that $I_1(k_2,k_4)=D_2(C^{21}(\alpha))^2=0$, where the values of α_a are the same as before.

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